

D2-D8 system with massive strings and the Lifshitz spacetimes

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Abstract

The Romans' type IIA supergravity allows fundamental strings to have explicit mass term at the tree level. We show that there exists a (F1,D2,D8) brane configuration which gives rise to $Lif_4^{(2)} \times R^1 \times S^5$ vacua supported by the massive strings. The presence of D8-branes naturally excites massive fundamental strings. A compactification on circle relates these Lifshitz massive type-IIA background with the axion-flux $Lif_4^{(2)} \times S^1 \times S^5$ vacua in ordinary type-IIB theory. The massive T-duality in eight dimensions further relates them to yet another $\widetilde{Lif}_4^{(2)} \times S^1 \times S^5$ vacua constituted by (F1,D0,D6) system in ordinary type IIA theory. The latter vacua after compactification to four dimensions generate two 'distinct' electric charges and a constant magnetic field, all living over 2-dimensional plane. This somewhat reminds us of a similar set up in quantum Hall systems.

1 Introduction

The AdS/CFT [1, 2, 3] holographic applications in strongly coupled quantum systems exhibiting nonrelativistic symmetries near critical points has been the focus of studies recently [4]-[24]. Some of the holographic applications may involve strongly coupled fermionic systems at finite density or a gas of ultra cold atoms [4, 5]. Some related framework of Galilean symmetry were studied even earlier [16]. For finite temperature properties such as phase transitions, transport and viscosity we include black-holes in asymptotically AdS backgrounds. For superconductivity phenomenon the 4-dimensional non-relativistic geometry generically involves spontaneously broken Higgs phase where the Maxwell field is massive [6, 9]. We shall discuss here some examples of 10-dimensional Lifshitz spacetimes where a Higgs phase instead involves 2-rank antisymmetric tensor field and has massive fundamental strings. The two phenomena indeed have parallel from 10-dimensional perspective. It is because a Kaluza-Klein compactification of a (massive) 2-rank tensor field on a circle gives rise to a (massive) gauge field in lower dimensions.

Our main interest in this work is to construct Lifshitz solutions with dynamical exponent $a = 2$, directly in Roman's type IIA supergravity [25]. The massive type IIA theory is the only known example of a 10-dimensional maximal supergravity where the string field is explicitly massive at the tree level. Thus the massive type IIA provides an unique setup to look for Lifshitz and Schrödinger like solutions which involve massive fields and study their dual nonrelativistic field theories on the boundary. There are no prior attempts to our knowledge where the same has been worked out for Romans type IIA supergravity, however $Sch_4^{(3)} \times S^6$ massive string vacua are known to us [17]. The $Lif_4^{(2)} \times R^1 \times S^5$ have been shown to exist in 10d type IIB string theory when appropriate axion flux is switched on [19]. We provide an explicit example of $Lif_4^{(2)} \times R^1 \times S^5$ background, which is a solution of massive type IIA and it does involve massive B -field. It is a background generated by a bound state of $(F1, D2, D8)$ brane system. It is important to know all such vacua as their compactification on $S^1 \times S^5$ immediately provides the prototype $Lif_4^{(2)}$ background in 4d, which is proposed to be dual to non-relativistic Lifshitz theory with dynamical exponent two. By exploiting massive T-duality symmetries in 8d we further provide another example of $\widetilde{Lif}_4^{(2)} \times S^1 \times S^5$ background, which is generated by massless $(F1, D0, D6)$ brane system in ordinary type IIA theory.

The paper is organised as follows. In the section-2 we first review the relevant aspects of massive type IIA sugra action and its known Schrödinger solution. In section-3 we write down new Lifshitz solution generated by the $(F1, D2, D8)$ system and discuss subtle aspects under massive T-duality for these solutions. The section-4 contains ordinary type IIA solution involving $(F, D0, D6)$ branes, which is supported by RR 2-form flux and massless B -field. They have a very crucial $(B_{(2)} \wedge G_{(2)})^2$

coupling between them. The string B -field gets essentially nested with constant magnetic flux of $G_{(2)}$ over $2d$ plane. We next discuss their consistent reduction to four and five dimensions and their effective bulk theories in section-5. We provide an M-theory uplift of these vacua in section-6. A summary is provided in the section-7.

2 $Sch_4^{(3)} \times S^6$: massive strings

In this section we mainly review some useful information about 10-dimensional massive type IIA supergravity [25] and a known Schrödinger vacua $Sch_4^{(3)} \times S^6$ [17]. One may choose to directly skip to the next section. The Romans type IIA theory is the only known maximal supergravity in ten dimensions which allows massive string B field. The theory is described by the following bosonic action

$$S = \frac{1}{G_N} \int \left[e^{-2\phi} \left(*R + 4(d\phi)^2 - \frac{1}{2}(H_{(3)})^2 \right) - \frac{1}{2}(G_{(2)})^2 - \frac{1}{2}(G_{(4)})^2 - *\frac{m^2}{2} \right] \quad (1)$$

where topological terms have been dropped because these would be vanishing for the Lifshitz backgrounds we shall be studying in this paper, see for details in [25, 31].¹ The field strengths are defined as

$$H_{(3)} = dB_{(2)}, \quad G_{(2)} = dC_{(1)} + mB_{(2)}, \quad G_{(4)} = dC_{(3)} + B_{(2)} \wedge dC_{(1)} + \frac{m}{2}B_{(2)} \wedge B_{(2)} \quad (2)$$

where m is the mass parameter and positive cosmological constant of the 10-dimensional theory. The cosmological constant however generates a nontrivial potential term for the dilaton field. Other than the nonsupersymmetric Freund-Rubin vacua in [25], some known BPS solutions of the theory include D8-branes [26, 27, 28, 29], the $K3$ compactifications [30], the $(D6, D8)$ and $(D4, D6, D8)$ bound states [31, 32]. Under the ‘massive’ T-duality [27] the D8 brane can be mapped over to a D7 brane in ordinary type IIB string theory. The string B -field is explicitly massive with mass square given by m^2 , and it plays an important role in obtaining nonrelativistic Schrödinger solutions [17]. The massive type IIA theory however never admits a flat Minkowski vacuum solution. But in the $m \rightarrow 0$ limit the massive theory reduces to ordinary type IIA supergravity.

An observed common feature in four-dimensional AdS gravity theories has been that in order to obtain Schrödinger or Lifshitz type non-relativistic solutions one needs to include massive (Proca) gauge fields in the action [4, 5]. (Although massless gauge fields can give rise to nonrelativistic vacua however, in simple cases of D-branes compactified along lightcone coordinate [20, 21], they usually give rise to conformal (or hyperscaling) Lifshitz or Schrödinger vacua.) Particularly for massive type IIA theory the existence of Schrödinger solution $Sch_4^{(3)} \times S^6$ has already been shown

¹ We are adopting a convention: $\int (H_{(p)})^2 = \int H_{(p)} \wedge *H_{(p)} = \frac{1}{p!} \int d^{10}x \sqrt{-g} H_{\mu_1 \dots \mu_p} H^{\mu_1 \dots \mu_p}$ and for scalar quantities like curvature scalar: $\int *R = \int d^{10}x \sqrt{-g} R$.

in [17]. In the rest of this section we review the $a = 3$ Schrödinger solution to familiarise ourselves, and also because these vacua are constituted by D0, D2, D8 branes, alongwith massive strings which we will encounter again when we write down Lifshitz solutions of the theory.

The massive type IIA action in the tensorial notation is given by

$$S = \frac{1}{G_N} \int d^{10}x \sqrt{-g} \left[R - \frac{1}{2}(\partial_\mu \phi)^2 - \frac{e^{-\phi}}{2 \cdot 3!} (H_{\mu\nu\lambda})^2 - \frac{e^{3\phi/2}}{2 \cdot 2!} (G_{\mu\nu})^2 - \frac{e^{\phi/2}}{2 \cdot 4!} (G_{\mu\nu\lambda\sigma})^2 - \frac{e^{5\phi/2}}{2} m^2 \right], \quad (3)$$

with the field strengths $H_{\mu\nu\lambda} = 3\partial_{[\mu} B_{\nu\lambda]}$, $G_{\mu\nu} = 2\partial_{[\mu} C_{\nu]} + mB_{\mu\nu}$, $G_{\mu\nu\lambda\sigma} = 4\partial_{[\mu} C_{\nu\lambda\sigma]} + 6B_{[\mu\nu} G_{\lambda\sigma]} - 3mB_{[\mu\nu} B_{\lambda\sigma]}$. The equations of motion of this theory admit following $Sch_4^{(3)} \times S^6$ solution [17]

$$\begin{aligned} ds^2 &= L^2 \left(-\frac{2}{z^6} (dx^+)^2 + \frac{-2dx^+ dx^- + dy^2 + dz^2}{z^2} + \frac{5}{2} d\Omega_6^2 \right), \\ e^\phi &= g_a, \quad C_{+-y} = -\sqrt{10} g_a^{-\frac{1}{4}} \frac{L^3}{3z^3}, \quad C_+ = -\frac{\sqrt{5}}{3} \frac{g_a^{-3/4} L}{z^3} \\ B_{+y} &= -\frac{g_a L^2}{2z^4}, \end{aligned} \quad (4)$$

where $L^2 = \frac{2}{m^2 g_a^{5/2}}$. But we should have string coupling $g_a \ll 1$ and $L \gg 1$. The radius of curvature of the sphere is directly related to the mass parameter. Note that the Lorentz invariance is explicitly broken in the solutions, although spin group of the sphere is intact. The Schrödinger vacua (4) involves a collection of D0, D2, and D8 branes (which are wrapped around S^6), along with massive fundamental strings (F1) stretched along y direction. The Einstein equations do involve a nontrivial stress-energy tensor component T_{++} that receives contributions from the lightlike components of these fields. From the solution we learn that in ten-dimensional sense the matter (dust) responsible for a Schrödinger solutions is made up of D-branes and most importantly the *massive* strings. The boundary of the spacetime is located at $z = 0$, and near to the boundary the metric and fields become divergent, which usually is the case for non-relativistic vacua with $a > 1$. But everything is fine in the interior of the spacetime including the curvature scalar which is a constant quantity.

3 $Lif_4^{(2)} \times R^1 \times S^5$ vacua

3.1 D2-D8: tied up with massive strings

It is known that the Freund-Rubin $AdS_4 \times S^6$ maximally symmetric vacua in Romans theory [25] is constituted by (D2, D8) branes. But there are no ‘massive’ B -field excitations in them. We show that one can construct Lifshitz vacua that

are supported by massive B -field, D2 and D8 branes. The strings in these solutions becomes massive after gobbling up all D0 branes, unlike the Schrödinger type vacua (4) above where some D0 branes are still present. Thus the D0 brane charges will not be explicitly seen in the following Lifshitz solutions. These higgsed Lifshitz solutions are given by (in string metric)

$$\begin{aligned} ds^2 &= \alpha' L^2 \left(-\frac{dt^2}{z^4} + \frac{dx_1^2 + dx_2^2}{z^2} + \frac{dz^2}{z^2} + \frac{dy^2}{q^2} + d\Omega_5^2 \right), \\ e^\phi &= g_a, \quad C_{(3)} = -\frac{(\alpha' L^2)^{\frac{3}{2}}}{g_a} \frac{1}{z^4} dt \wedge dx_1 \wedge dx_2, \\ B_{(2)} &= \frac{\alpha' L^2}{q z^2} dt \wedge dy, \end{aligned} \quad (5)$$

with $L = \frac{2}{g_a m l_s}$, where m being the mass parameter in the Romans' action. The q is free (length) parameter and g_a is perturbative string coupling in this massive type IIA vacuum. Note L , which is dimensionless, determines overall radius of curvature of the 10-dimensional spacetime. While m being a parameter in the lagrangian and it determines L , therefore Romans' theory with $m \ll \frac{2}{g_a l_s}$ would be preferred so that we can have $L \gg 1$ in the solutions (5), else we cannot trust these classical vacua.²

The Lifshitz configuration (5) describes a parallel stack of D2-branes stretched along (x_1, x_2) directions and 'massive' fundamental strings that are aligned along y direction. The D8-branes wrap around S^5 completely while remaining stretched along the patch (x_1, x_2, y) . The D2 charges can be found as

$$Q_{D2} \sim \frac{1}{\omega_5} \int e^{\frac{\phi}{2}} \star G_{(4)} \sim \frac{4 l_s^5 L^5 l_y}{g_a q} \propto N \quad (6)$$

where l_y is coordinate size of y and ω_5 is the unit volume of 5-sphere. Thus eq.(5) describes a $(F1, D2, D8)$ configuration, in which D2 branes are stacked inside D8 worldvolume. The D2 stack is studded (threaded) by massive F-strings, having mass $m = \frac{2}{L g_a l_s}$. The D0 branes are all gauged away (or eaten up) by the F-strings which have become massive. This phenomenon happens due to the higgsing (or stueckelberg mechanism) in massive type IIA theory [25]. In these solutions y remains an overall isometry direction which may be compact also. The holographic z coordinate acts as a common transverse direction for all the branes.

3.2 $Lif_4^{(2)} \times S_w^1 \times S^5$

The 10-dimensional string coupling g_a has to be weak so that we can trust the Lifshitz vacua (5). The small g_a however tends to send the mass of F-strings to

²Note, from the D8 brane/domain wall correspondence [27], one typically expects $m \approx \frac{g_a N_{D8}}{l_s}$, a value which is definitely well within $\frac{2}{g_a l_s}$.

higher values, but for sufficiently large $L \sim O(\frac{1}{g_a^2})$ it can support massive strings in the background (note the lighter ones with $m \approx \frac{q_a}{l_s}$ only need to be excited). So far we considered y as noncompact coordinate and q an associated arbitrary length scale. Let us consider the case when y is a compact circle, *i.e.* $y \sim y + 2\pi R_y$. The physical radius of y -circle will be $\frac{l_s L R_y}{q}$. Therefore for smaller q values the massive strings can indeed be excited in the transverse y direction of the D2-branes. It would also be appropriate here to take $q \equiv w R_y$, with w being an integer. (It could also be the wrapping number of D8-branes. By wrapping number here we mean by number of times a single D8 brane wraps around S^1 .) The vacua (5) will now be described as $Lif_4^{(2)} \times S_w^1 \times S^5$:

$$ds^2 = \alpha' L^2 \left(-\frac{dt^2}{z^4} + \frac{dx_1^2 + dx_2^2}{z^2} + \frac{dz^2}{z^2} + \frac{1}{w^2} d\psi^2 + d\Omega_5^2 \right),$$

$$e^\phi = g_a, \quad C_{(3)} = -\frac{(\alpha' L^2)^{\frac{3}{2}}}{g_a z^4} dt \wedge dx_1 \wedge dx_2, \quad B_{(2)} = \frac{\alpha' L^2}{w z^2} dt \wedge d\psi, \quad (7)$$

where $0 \leq \psi \leq 2\pi$. (ψ can also be viewed as an orbifolded circle.) Anyhow $w > 1$ tells us about the comparative sizes of S^1 and S^5 . Due to this size difference a D8-brane can wrap S^1 w -times more as compared to its wrapping around S^5 . Especially for $w = 1$, both S^1 and S^5 will have the same size. But small w is always preferred in (7). For large w (or q) the radius of S^1 will become sub-stringy, it would then be appropriate to switch over to the T-dual vacua in type IIB string theory. We would discuss it in the next section.

The Lifshitz solutions (5) or (7) both have following asymmetric scaling symmetry

$$t \rightarrow \lambda^2 t, \quad x^i \rightarrow \lambda x^i, \quad z \rightarrow \lambda z \quad (8)$$

Note y (or ψ) coordinate is not required to scale at all and it is one of the characteristics of the $a = 2$ Lifshitz vacua.

3.3 Massive 9d T-duality

Upon S^1 compactification the Romans theory gives rise to a massive supergravity in nine dimensions. Keeping only the fields relevant for our background (5), (a complete circle compactification can be found [27]) a 9d massive action can be written as

$$S \sim \int d^9 x \sqrt{-g} e^{-2\phi_9} \left[R + 4(\partial_\mu \phi_9)^2 - \frac{1}{2}(\partial_\mu \rho)^2 - \frac{e^{\sqrt{2}\rho}}{2.2!} (F_{\mu\nu})^2 - \frac{e^{\frac{\rho}{\sqrt{2}}}}{2} m^2 (A_\mu)^2 \right. \\ \left. - \frac{e^{\frac{-\rho}{\sqrt{2}}}}{2.4!} (G_{\mu\nu\lambda\sigma})^2 - \frac{e^{\frac{-\rho}{\sqrt{2}}}}{2} m^2 \right], \quad (9)$$

where the vector field A_μ which arises directly from reduction of massive tensor component $\hat{B}_{\mu y}$ is only kept in the action. The 9d dilaton field is given by

$$\phi_9 = \hat{\phi} - \frac{1}{2\sqrt{2}} \rho$$

whereas

$$\rho = \frac{1}{\sqrt{2}} \log(\hat{g}_{yy})$$

defines the radion mode along y . (The $10d$ fields are denoted with a hat sign to distinguish them from 9-dimensional ones.) The Kaluza-Klein gauge field is set to vanish. The topological terms in the action are also ignored as these are not relevant for the Lifshitz background we are studying here. By setting $\alpha' = 1$, after compactification along y the eq.(5) reduces to

$$\begin{aligned} ds_{9d}^2 &= L^2 \left(-\frac{dt^2}{z^4} + \frac{dx_1^2 + dx_2^2 + dz^2}{z^2} + d\Omega_5^2 \right), \\ e^{\phi_9} &= \sqrt{\frac{q}{L}} g_a, \quad C_{(3)} = -\frac{L^3}{g_a z^4} dt \wedge dx_1 \wedge dx_2 \\ A_{(1)} &= \frac{L^2}{q z^2} dt, \quad e^{-\sqrt{2}\rho} = q^2 L^{-2}, \end{aligned} \quad (10)$$

It can be checked that (10) is a consistent solution of the $9d$ action (9). Actually both the dilaton ϕ_9 and the ρ have constant background values. Since it is a solution of $9d$ type II sugra with a massive vector field, the above $9d$ Lifshitz vacua can be uplifted back to 10-dimensional type IIB theory (over a dual radius circle) by exploiting ‘massive’ T-duality [27]. By employing the massive duality rules [27, 33] we get following type IIB solution

$$\begin{aligned} ds_{IIB}^2 &= L^2 \left(\tilde{q}^2 d\tilde{y}^2 - \frac{2\tilde{q} dt d\tilde{y}}{z^2} + \frac{dx_1^2 + dx_2^2}{z^2} + \frac{dz^2}{z^2} + d\Omega_5^2 \right) \\ G_{(5)} &= \frac{1}{\sqrt{2}g_b} \left(\frac{4\tilde{q}L^4}{z^5} dt \wedge dx_1 \wedge dx_2 \wedge d\tilde{y} \wedge dz + \text{self dual} \right) \\ e^{\phi_b} &= g_b, \quad \chi = \frac{2\tilde{q}}{g_b} \tilde{y}. \end{aligned} \quad (11)$$

The type IIB string coupling g_b , radius of curvature L , and flux \tilde{q} are all independent parameters. The coupling g_b should be kept small. The parameter \tilde{q} determines the required axion flux and also the relative size of \tilde{y} in the metric. (Recall an ordinary type IIB theory has no mass scale of its own unlike the Romans’ type IIA theory.) The massive $9d$ T-duality does indeed relate \tilde{q} with Romans’ mass through

$$g_b = g_a \frac{q}{L}, \quad m = \frac{2}{g_a L} \equiv \frac{2\tilde{q}}{g_b}, \quad \tilde{q} \equiv \frac{q}{L^2} \quad (12)$$

Note the flux parameter \tilde{q} has mass dimension one. The vacua (11) describes Lifshitz solution of type IIB theory first obtained by [19], if we redefine the coordinates as $\tilde{y} = x^+, t = x^-$.

Note that the axion field has a linear dependence on the circle coordinate ($\tilde{y} \sim \tilde{y} + 2\pi\tilde{R}_y$), where $\tilde{R}_y = \frac{\alpha' L^2}{R_y}$ via T-duality. Due to this the axion undergoes discrete jumps (determined by parameter \tilde{q}) each time it goes around the circle, as it has to be periodic. One should take $\tilde{q} = \frac{w R_y}{L^2}$ with an integral w . The w will now effectively count the axions (or the number of D7 branes)

$$Q_{D7} = \frac{1}{2\pi} \int d\chi = \tilde{q}\tilde{R}_y = w . \quad (13)$$

The D3 charge is given by

$$Q_{D3} \sim \frac{1}{\omega_5} \int \star G_{(5)} = \frac{2\sqrt{2}L^4}{g_b} \approx N \quad (14)$$

taking $L^4 \approx g_b N$. Thus the eq.(11) describes a bound state of $(W, D3, D7)$ system with wave. The coordinates z and \tilde{y} together constitute two transverse directions of D7 branes. It is the same kind of effect which the massive T-duality brings under which D8-brane of massive type IIA is mapped over to D7-brane in type IIB and vice-versa [27]. Thus we have confirmed that the Romans type IIA theory with massive B -field admits $(F1, D2, D8)$ Lifshitz vacua and it also provides a consistent T-dual description of the $(W, D3, D7)$ Lifshitz vacua of type IIB string theory.

The vacua presented here are different from other Lifshitz vacua obtained in $6d$ and $5d$ gauged/massive supergravities with arbitrary dynamical exponents in [22, 23]. Those $Lif_4^{(a)} \times H_2$ and $Lif_3^{(a)} \times H_2$ vacua with $a > 1$ (the H_2 being sphere, hyperboloid or flat space) can also be lifted to $10d$, but these are different cases. Only dilatonic scalars are present in those gauged/massive sugra theories. We are not sure if they could be related to $(F1, D2, D8)$ background.

3.4 $5d$ massive tensor model

After doing an explicit S^5 compactification the 10-dimensional background (5) reduces to the following 5-dimensional $Lif_4^{(2)} \times R^1$ solution (we set $\alpha' = 1$ for simplicity)

$$ds^2 = L^2 \left(-\frac{dt^2}{z^4} + \frac{dx_1^2 + dx_2^2}{z^2} + \frac{dz^2}{z^2} + \frac{dy^2}{q^2} \right) \\ e^{\phi_5} = g_5, \quad \bar{C}_{(3)} = -\frac{L^3}{g_5 z^4} dt \wedge dx_1 \wedge dx_2, \quad B_{(2)} = \frac{L^2}{q z^2} dt \wedge dy, \quad (15)$$

where $g_5 \equiv \frac{g_a}{L^{5/2}}$ is the $5d$ string coupling constant. The g_a is 10-dimensional coupling. The corresponding $5d$ effective action can be presented as

$$S_{5d} \sim \int \left[e^{-2\phi_5} (*R + 4(d\phi_5)^2 - \frac{1}{2}(dB_{(2)})^2 + *\frac{20}{L^2}) - \frac{\bar{m}^2}{2}(B_{(2)})^2 - \frac{1}{2}(d\bar{C}_{(3)})^2 - *\frac{\bar{m}^2}{2} \right] (16)$$

where the parameters are related as $\bar{m} = L^{\frac{5}{2}} m \equiv \frac{1}{g_5 L}$. The B field is explicitly massive. One may however rewrite the above action in terms of an axion field after the Hodge duality, $*dC_{(3)} = dC_{(0)}$. The background (15) is an exact solution of the action (16) which has two cosmological constant terms of opposite signs.

3.5 The 4d Proca model

After explicit compactification on $S^1 \times S^5$ the 10-dimensional background (5) reduces to the following 4-dimensional $Lif_4^{(2)}$ solution

$$\begin{aligned} ds^2 &= \bar{L}^2 \left(-\frac{dt^2}{z^4} + \frac{dx_1^2 + dx_2^2}{z^2} + \frac{dz^2}{z^2} \right) \\ e^{\phi_4} &= g_4, \quad \mathcal{A}_{(1)} = -\frac{L}{z^2} dt, \end{aligned} \quad (17)$$

where the curvature radius $\bar{L} \equiv \frac{L}{g_4}$ and $g_4 \equiv \frac{g_a \sqrt{q}}{L^3}$ is the 4d string coupling constant. The dilaton has got constant background value. This background is a solution of the following Einstein-Proca effective action, which directly follows from $S^1 \times S^5$ compactification of the Romans theory,

$$S_{4d} \sim \int \left[*R - 2(d\phi_4)^2 - \frac{e^{-2\phi_4}}{2} (d\mathcal{A}_{(1)})^2 - \frac{m_4^2 e^{2\phi_4}}{2} (\mathcal{A}_{(1)})^2 + * \frac{20e^{2\phi_4}}{L^2} - * \frac{5m_4^2 e^{4\phi_4}}{2} \right] \quad (18)$$

where new mass parameter $m_4 = \frac{2}{g_4 L}$. The action includes a Proca field (*descending from the winding modes of the massive B-field in (5)*). If we plug in the constant value of the dilaton g_4 , the action (18) further simplifies to

$$S_{4d} \sim \int \left[*R - \frac{1}{2g_4^2} (d\mathcal{A}_{(1)})^2 - \frac{2}{L^2} (\mathcal{A}_{(1)})^2 + * \frac{10g_4^2}{L^2} \right]. \quad (19)$$

We remind that the cosmological constant and Proca mass are very precisely related in the action so as to admit Lifshitz $a = 2$ vacua (17). However the action (19) cannot have Schrödinger $a = 3$ spacetime as solutions. Instead another Einstein-Proca action with a different cosmological constant, that follows from S^5 compactification of the fields in (4), will allow $Sch_4^{(3)}$ solutions, see for the details [17, 18]. From this example we realise that, although 4d actions differ in only the value of cosmological constants, but their 10d solutions have entirely different matter field contents. These massive gauge-gravity field models have been studied as effective gravity models describing holographic superconductor phenomena on 3-dimensional boundary [5, 4, 7]. We shall provide alternative 4d Lifshitz models which have two Maxwell potentials and they are obtainable by exploiting massive T-duality in the forthcoming sections.

4 T^2 dual of $(F1, D2, D8)$ system

The 8-dimensional type II supergravity has a T-duality group $SL(2, R) \times SL(2, R)$ including the massive version of supergravity [31]. Under this massive duality symmetry the mass parameter m (or dual 10-form $G_{(10)}$ flux) in Romans theory compactified on T^2 gets mapped into $G_{(2)}$ -flux (along 2-torus) in ordinary type-IIA

compactified on T^2 . Using this massive duality we would like to map $Lif_4 \times R^1 \times S^5$ vacua (5) into the $G_{(2)}$ -flux vacua of ordinary type IIA theory.

Let us choose x_1 and x_2 to be the coordinates along T^2 . The RR 2-form flux corresponds to having a nonvanishing constant magnetic component

$$G_{flux} \equiv m dx_1 \wedge dx_2 = \frac{2}{g_a L} dx_1 \wedge dx_2 \quad (20)$$

filling the entire T^2 . (Note we must take mass m exactly equal to $\frac{2}{g_a L}$ as it is fixed earlier.) Using the duality map worked out in [31], we can now write down corresponding vacua of ordinary type IIA theory,

$$\begin{aligned} ds^2 &= L^2 \left(-\frac{dt^2}{z^4} + \frac{z^2}{L^4} (dx_1^2 + dx_2^2) + \frac{dz^2}{z^2} + \frac{dy^2}{q^2} + d\Omega_5^2 \right), \\ e^\phi &= \frac{z^2}{L^2} g_a, \quad G_{(2)} = \frac{4L^3}{z^5 g_a} dz \wedge dt + G_{flux}, \\ B_{(2)} &= \frac{L^2}{qz^2} dt \wedge dy, \end{aligned} \quad (21)$$

Thus it could see that corresponding RR 1-form gauge field has a most general form

$$C_{(1)} = C_0 dt + C_1 dx^1 + C_2 dx^2, \quad C_0 = -\frac{L^3}{g_a z^4}, \quad C_1 = -\frac{1}{g_a L} x_2, \quad C_2 = \frac{1}{g_a L} x_1 \quad (22)$$

which has both electric as well as magnetic components. However no $C_{(3)}$ background is present, that is because after the duality all D2 branes (of massive-IIA) now reappear as D0 branes in (21). The B field being *massless*, as it is in ordinary type IIA, however interacts (or gets nested) with nontrivial $G_{(2)}$ flux of D6-branes in the effective action. (Note all D8 branes morph into D6 branes after massive duality on T^2). The B equation of motion has a contributions from the interaction term in 4-form field strength $G_{(4)} = dC_{(3)} + G_{(2)} \wedge B_{(2)}$ in type IIA, which for the above background (21) contributes a term like $\sim \frac{1}{L^2} (B_{ty})^2$ in the action due to nonvanishing $G_{(2)}$ flux. The equation of motion of $C_{(3)}$ is trivially satisfied for $C_{(3)} = 0$. Thus $G_{(2)}$ has got both electric (D0) and magnetic (D6) components all aligned along noncompact directions in $5d$ spacetime in the above. Thus the Lifshitz background (21) essentially represents $(F1, D0, D6)$ bound state in ordinary type IIA. *The D0 branes have reappeared back and F-strings are now massless. This maybe called as un-higgsing phenomenon carried out by massive duality on T^2 .*

5 Lower dimensional models

5.1 4d Lifshitz theory and a quantum Hall system

Let us now consider the compact case where $y \sim y + 2\pi R_y$ in (21). The D6-branes will wrap around $S^1 \times S^5$ completely and their G_2 flux would fill entire $x_1 - x_2$

plane. The massless F1-strings will be wrapping around y circle. The $4d$ string coupling goes to vanishing value in the UV (as $z \rightarrow 0$). Hence these vacua after compactification give rise to $\widetilde{Lif}_4^{(2)}$ vacua (in string metric)

$$\begin{aligned} ds^2 &= -\frac{L^2 dt^2}{z^4} + \frac{z^2}{L^2}(dx_1^2 + dx_2^2) + \frac{L^2 dz^2}{z^2}, \\ e^{\phi_4} &= g_a \frac{\sqrt{q}}{L^3} \frac{z^2}{L^2}, \quad C_{(1)} = \frac{1}{g_a} \left(-\frac{L^3}{z^4} dt - \frac{x_2}{L} dx_1 + \frac{x_1}{L} dx_2 \right), \\ \mathcal{A}_{(1)} &= \frac{-L^2}{qz^2} dt, \end{aligned} \tag{23}$$

with two distinct Maxwellian gauge fields \mathcal{A} and C . The scaling property of the solution (23) under the $z \rightarrow \lambda z$ is

$$\begin{aligned} t &\rightarrow \lambda^2 t, \quad x_1 \rightarrow \lambda^{-1} x_1, \quad x_2 \rightarrow \lambda^{-1} x_2, \\ g_{\mu\nu} &\rightarrow g_{\mu\nu}, \quad e^\phi \rightarrow \lambda^2 e^\phi, \quad C \rightarrow \lambda^{-2} C, \quad \mathcal{A} \rightarrow \mathcal{A} \end{aligned} \tag{24}$$

It has a dynamical exponent of time as $a = 2$, but crucially has ‘negative’ scaling exponent ($a_x = -1$) for spatial directions x_1 and x_2 . It is quite plausible because D0 branes are delocalised over x_1, x_2 plane as well as there is nontrivial magnetic flux G_{12} . (The negative scaling exponent of spatial coordinates is usually associated with negative pressure along those direction in the CFT.)³

The background (23) is unique Lifshitz vacuum in the sense that it describes two ‘electrically’ charged objects interacting with a magnetic flux. It can be checked that the Lifshitz vacua (23) is indeed a solution of following massless $4d$ effective action

$$S_{4d} \sim \int \left[e^{-2\phi_4} \left(*R + 4(d\phi_4)^2 - \frac{q^2}{2L^2} (d\mathcal{A}_{(1)})^2 + * \frac{20}{L^2} \right) - \frac{L^6}{2q} G_{(2)}^2 - \frac{qL^4}{2} (\mathcal{A}_{(1)} \wedge G_{(2)})^2 \right] \tag{25}$$

where ϕ_4 is 4-dimensional dilaton field and field strength $G_{(2)} = dC_{(1)}$. Note that the scaling of the fields in (24) gives rise to following property of the action:

$$S_{4d} \rightarrow \frac{S_{4d}}{\lambda^4}. \tag{26}$$

One thing to observe is that there is no mass term for gauge fields in the action (25). However two Maxwell fields in the action have Chern-Simons like interaction

³The nonrelativistic phenomenon is expressed by the hamiltonian $H = \frac{(\vec{p})^2}{2m} + \dots$, and the scaling property (24) implies that the quasi-particle masses must scale as $m \rightarrow \lambda^4 m$. Hence the particle mass (number) increases when we probe bigger length scales (or at low energy). It should be plausible for a quantum system exhibiting negative dynamical exponent for spatial coordinates.

between them. Hence they give rise to two types of charged objects with a Chern-Simons like interaction $(\mathcal{A} \wedge dC)^2$ between them. The boundary Lifshitz theory lives over 2-dimensional spatial plane. By constant scaling of the fields the action (25) can be brought to a canonical form

$$S_{4d} \sim \int \left[*R - 2(d\phi_4)^2 - \frac{e^{-2\phi_4}}{2}(d\mathcal{A}_{(1)})^2 + *\frac{20}{L^2}e^{2\phi_4} - \frac{1}{2}(dC_{(1)})^2 - \frac{e^{-2\phi_4}}{2}(\mathcal{A}_{(1)} \wedge dC_{(1)})^2 \right] \quad (27)$$

and corresponding 4d solution also can be written as ⁴

$$\begin{aligned} ds^2 &= L^2 \left(-\frac{dt^2}{z^4} + \frac{z^2}{L^4}(dx_1^2 + dx_2^2) + \frac{dz^2}{z^2} \right), \\ e^{\phi_4} &= g_0 \frac{z^2}{L^2}, \quad C_{(1)} = \frac{1}{g_0} \left(-\frac{L^3}{z^4} dt - \frac{x_2}{L} dx_1 + \frac{x_1}{L} dx_2 \right), \\ \mathcal{A}_{(1)} &= -\frac{L}{z^2} dt, \end{aligned} \quad (28)$$

where $g_0 (\equiv g_a \frac{\sqrt{q}}{L^3})$ is 4d coupling constant. The parameter q has altogether disappeared from the solutions (28) and it only determines the coupling g_0 . Since the dilaton runs towards strong coupling in the IR, this solution gives valid CFT description only in the $z < L$ (near UV region). In the UV region the Lifshitz theory becomes almost free.

It is tempting to holographically relate the bulk theory (27) to some known nonrelativistic condensed matter phenomenon in a plane on the boundary, involving two distinct (electrical) charges interacting in presence of constant magnetic field (or current), somewhat like in quantum Hall effect. There are no mass terms for gauge fields hence the solution (28) cannot describe superconducting or higgs phase. For the bulk solution (28), the gauge interaction terms in the action (27) is very crucial, but it is of rather unusual type to motivate. However it is not uncommon for such tensorial interactions to arise in bulk gravity (string) theory. The massless gauge-gravity action (27) certainly represents a new critical phenomenon, but it is also related via ‘massive’ T-duality to another action having massive gauge fields (19) and describing superconducting phenomenon [4, 7].

Next the gauge field \mathcal{A} specially couples to the dilaton hence it is distinct as compared to the other gauge field C which is totally decoupled from dilaton in (27). The former may thus have its origin in the Hall (excitations) carriers. As we understand from the quantum Hall effect that, there is a constant magnetic field uniformly spread over a plane alongwith an electric field (EMF) applied in one (say x_1) direction of the plane. Consequently a quantized Hall voltage (current) gets

⁴ The 4d string metric has a constant negative curvature. It does not include curvature singularity.

generated along another (here x_2) direction of the planar system. From this analogy we understand that \mathcal{A}_0 can possibly be the source of Hall charges. In holography, the boundary value of the time-component of bulk gauge field represents charge sources in boundary. Furthermore, \mathcal{A}_0 is essentially a component of string B -field (wrapped on y circle). Since $L^4 \sim O(N) \gg 1$ and L is an important overall parameter. We find that the two gauge potentials behave as

$$\mathcal{A}_0 \sim O(N^{\frac{1}{4}}), \quad C_0 \sim O(N^{\frac{3}{4}}) \quad (29)$$

Thus their respective carrier concentrations would have $O(N^{\frac{1}{2}})$ difference. However if we compare respective Lorentz invariants in the action involving electric and magnetic fields, then

$$\begin{aligned} e^{-2\phi_4}(\partial_z \mathcal{A}_0)^2 &\sim -\frac{4g_0^2}{L^6} z^4, \\ (\partial_z C_0)^2 &\sim -\frac{16g_0^2}{L^6} z^4, \\ (\partial_i C_j)^2 &\sim \frac{4g_0^2}{L^6} z^4. \end{aligned} \quad (30)$$

It shows that, though the electric field contribution of $(\partial \mathcal{A})^2$ is comparatively weaker by a factor of $1/N$, but in the lagrangian (27) all gauge terms contribute with equal strengths due to varied dilatonic couplings. These on-shell quantities do not diverge near the boundary. Furthermore, in the bulk solution (28) we can in fact choose a gauge such that C_0, C_1 components are only nonvanishing alongwith \mathcal{A}_0 . This implies existence of two independent sources of charge (EMF) and an indepedent electric current in the boundary theory.

5.2 Alternative vacua with negative dynamical exponent

It is obvious that the action (27) would allow following Lifshitz vacua where dynamical exponent of time is instead negative while two spatial coordinates scale positively. Let us define a new holographic coordinate as

$$u^2 = \frac{L^4}{z^2} \quad (31)$$

and by scaling $t \rightarrow L^4 t$, we obtain from (28) a new kind of situation described by

$$\begin{aligned} ds^2 &= L^2 \left(-u^4 dt^2 + \frac{1}{u^2} (dx_1^2 + dx_2^2) + \frac{du^2}{u^2} \right), \\ e^{\phi_4} &= g_0 \frac{L^2}{u^2}, \quad C_{(1)} = \frac{1}{g_0} \left(-\frac{u^4}{L} dt - \frac{x_2}{L} dx_1 + \frac{x_1}{L} dx_2 \right), \\ \mathcal{A}_{(1)} &= -u^2 L dt, \end{aligned} \quad (32)$$

which is valid in the $u > L$ (IR) region. This new looking solution (32) has negative dynamical exponent for time ($a = -2$), such that

$$\begin{aligned} u &\rightarrow \lambda u, \quad t \rightarrow \lambda^{-2} t, \quad x_1 \rightarrow \lambda x_1, \quad x_2 \rightarrow \lambda x_2, \\ g_{\mu\nu} &\rightarrow g_{\mu\nu}, \quad e^{\phi_4} \rightarrow \lambda^{-2} e^{\phi_4}, \quad C \rightarrow \lambda^2 C, \quad \mathcal{A} \rightarrow \mathcal{A}. \end{aligned} \quad (33)$$

Note that the string coupling tends to blow up at shorter length scales (UV), while at longer (IR) scales it becomes almost a free theory. Thus the Lifshitz background (32) is suitable for describing a low energy Lifshitz theory at large length scales $u > L$. (While at shorter scales, $z < L$, the previous background (28) is more suitable.) In the present case the boundary field theory may actually describe electrodynamics because coupling remains weaker at longer distances.⁵ Once again there are two kinds of (electric) charges interacting with a constant magnetic field such as in quantum Hall systems near criticality.

5.3 5d vortex model: $B \wedge G$ interaction

A 5d effective action can be obtained by compactification of type IIA theory on the product spacetime like $\mathcal{M}_5 \times S^5$. We allow fields to have dependence only on \mathcal{M}_5 coordinates, such as we obtained in the background (21). Upon consistent truncations, and keeping only the relevant field content describing the equation (21), we get to following 5d action

$$S_5 \sim \int \left[e^{-2\phi_5} \left(*R + 4(d\phi_5)^2 - \frac{1}{2}H_{(3)}^2 + *\frac{20}{L^2} \right) - \frac{1}{2}G_{(2)}^2 - \frac{1}{2}(B_{(2)} \wedge G_{(2)})^2 \right] \quad (34)$$

The action has second rank tensor with field strength $H_{(3)} = dB_{(2)}$ and a vector field with $G_{(2)} = dC_{(1)}$, both interacting through a $B \wedge G$ like coupling. This interaction term will be important when string-like excitations (or flux tubes) couple with constant magnetic field in a transverse $2d$ plane in $4d$ CFT. As an example, the equation of motion of the action (34) are immediately solved by the following solution

$$\begin{aligned} ds^2 &= -\frac{L^2 dt^2}{z^4} + \frac{L^2 dy^2}{q^2} + \frac{z^2}{L^2}(dx_1^2 + dx_2^2) + \frac{L^2 dz^2}{z^2}, \\ e^{\phi_5} &= g_0 \frac{z^2}{L^2}, \quad G_{(2)} = \frac{4L^3}{g_0 z^5} dz \wedge dt + \frac{1}{g_0 L} dx_1 \wedge dx_2, \\ B_{(2)} &= -\frac{L^2}{q z^2} dt \wedge dy, \end{aligned} \quad (35)$$

⁵ The nonrelativistic phenomenon is expressed by the hamiltonian $H = \frac{(\vec{p})^2}{2m} + \dots$, and the scaling property (33) implies that the quasi-particle masses must scale as $m \rightarrow m/\lambda^4$. That is the quasi particle mass (number) get reduced when probed at large length scales (or with low energy) which appears to be plausible in a quantum system exhibiting negative dynamical exponent (for time) and having running coupling.

where $g_0 = g_a/\sqrt{L^5}$ is 5d coupling constant and q is arbitrary having dimensions of length. The 5d spacetime has constant negative curvature. The coupling remains weak in the UV region. The solution represents an uniform string like excitation extended in y direction and a constant magnetic G -flux $\sim \frac{1}{g_0 L}$ in the transverse $x_1 - x_2$ plane.

The above solution (35) is however well behaved only in the UV region where $z < L$. But, by using the transformation (31) one can transform it into the IR region ($u > L$)

$$\begin{aligned} ds^2 &= L^2 \left(-\frac{dt^2}{u^{-4}} + \frac{dx_1^2 + dx_2^2}{u^2} + \frac{du^2}{u^2} + \frac{dy^2}{q^2} \right) \\ e^{\phi_5} &= g_0 \frac{L^2}{u^2}, \quad G_{(2)} = \frac{4u^3}{g_0 L} du \wedge dt + \frac{1}{g_0 L} dx_1 \wedge dx_2, \\ B_{(2)} &= -\frac{L^2 u^2}{q} dt \wedge dy, \end{aligned} \quad (36)$$

where coupling remains weaker in the deep IR region. The dynamical exponent of time is given by $a = -2$. The charge sources are also present because the time-component C_0 is nontrivial. There is a constant magnetic background too, but there are also extended string (vortex) like objects present which couple to B_{0y} . It would be worthwhile to do a detailed analysis of the nonrelativistic CFT on the boundary. We hope to come back to this topic in a future investigation.

6 (W,M2,M7) brane system

The (F1,D2,D8) system (5) cannot be uplifted to M-theory at strong coupling as it is a solution of Romans theory for which we do not have 11-dimensional interpretation. Also because $L \sim \frac{1}{mg_a l_s} \rightarrow 0$ at strong coupling the spacetime becomes highly curved. On the other hand, we obtained (F1,D0,D6) in eq.(21) as the solution of ordinary type IIA supergravity. For the massless type IIA theory there exists an M-theory description [34]. For the vacua (21) the string coupling becomes strong in the IR region ($z \rightarrow \infty$) which usually is the case for CFT₃ (in relativistic cases also) [35]. At strong coupling a natural explanation of the theory should be found by studying these solutions in M-theory (with $R_{11} = g_a^{2/3} l_p$, $l_p = g_a^{-1/3} l_s$)

$$ds_{11d}^2 = e^{\frac{4\phi}{3}} (dx_{11} + A)^2 + e^{\frac{-2\phi}{3}} ds_{10d}^2 \quad (37)$$

where $x_{11} \sim x_{11} + 2\pi R_{11}$ is the eleventh coordinate. Correspondingly 11-dimensional supergravity vacua can be described as (setting $L = 1$ for simplicity)

$$\begin{aligned} ds_{11d}^2 &= \frac{1}{z^{4/3}} \left(-\frac{dt^2}{z^4} + z^2(dx_1^2 + dx_2^2) + \frac{dz^2}{z^2} + z^4(dx_{11} + \omega)^2 + \frac{1}{w^2} d\psi^2 + d\Omega_5^2 \right), \\ C_{(3)} &= \frac{1}{w z^2} dt \wedge dx_{11} \wedge d\psi, \end{aligned} \quad (38)$$

where fiber 1-form is

$$\omega_{(1)} = -z^{-4}dt - x_2dx_1 + x_1dx_2.$$

The background in (37) represents a conformal $Lif_5^{(2)} \times S_w^1 \times S^5$ vacua in M-theory which includes circle x_{11} fibered over a Lif_4 base geometry: $-\frac{dt^2}{z^4} + z^2(dx_1^2 + dx_2^2) + \frac{dz^2}{z^2}$. The fiber 1-form is special in that it has both time as well as magnetic components. The latter will have interpretation as M7-branes which entirely wrap $S_w^1 \times S^5$ and the x_{11} circle. (The M7-branes are the uplift of the D6-branes). The M2-branes also wrap (x_{11}, ψ) and their world-volume remains parallel to M7 branes. There is large anisotropy along x_1, x_2, x_{11} directions of the Lifshitz space;

$$t \rightarrow \lambda^2 t, \quad x_1 \rightarrow \lambda^{-1} x_1, \quad x_2 \rightarrow \lambda^{-1} x_2, \quad x_{11} \rightarrow \lambda^{-2} x_{11}, \quad (39)$$

under which the metric conformally scales as $g_{\mu\nu} \rightarrow \lambda^{-4/3} g_{\mu\nu}$ while 3-form $C \rightarrow \lambda^{-2} C$.

7 Summary

We have shown that for the Romans' type IIA supergravity the $(F1, D2, D8)$ brane configuration gives rise to the Lifshitz vacua $Lif_4^{(2)} \times R^1 \times S^5$. For this Lifshitz solution the fundamental strings have to be massive with $m = \frac{1}{g_a L l_s}$, and such that $L \gg 1$ and $g_a \ll 1$. A compactification of the massive type IIA theory on S^1 relates 9-dimensional $Lif_4^{(2)} \times S^5$ background with corresponding Scherk-Schwarz reduced type IIB Lifshitz vacua compactified on a dual circle. This duality map is known as 'massive' T-duality [27]. It has been known in literature that $Lif_4^{(2)} \times S^1 \times S^5$ spacetime is constituted by axion-flux $(W, D3, D7)$ system in type IIB string theory. The presence of D7-branes primarily requires constant axion flux switched on along S^1 in these solutions.

Further, using 'massive' T-duality symmetry in eight dimensions, we have related the Lifshitz vacua in Romans theory to yet another $\widetilde{Lif}_4^{(2)} \times R^1 \times S^5$ vacua which is constituted by $(F1, D0, D6)$ brane system of ordinary type IIA theory. But these latter vacua have negative dynamical exponents along two CFT directions. The interesting observation is that this $(F1, D0, D6)$ solution upon explicit compactification along $S^1 \times S^5$ gives rise to 4d Lifshitz vacua with two distinct types of electric charges supported by a constant magnetic flux (or current), all entirely living over $2d$ plane at the boundary. We speculate that this may represent phenomena akin to quantum Hall systems. We also have obtained the M-theory uplift of the $a = 2$ Lifshitz solutions.

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